national accelerator laboratory

TM-167
0320

BOOSTER MAGNET TOLERANCES

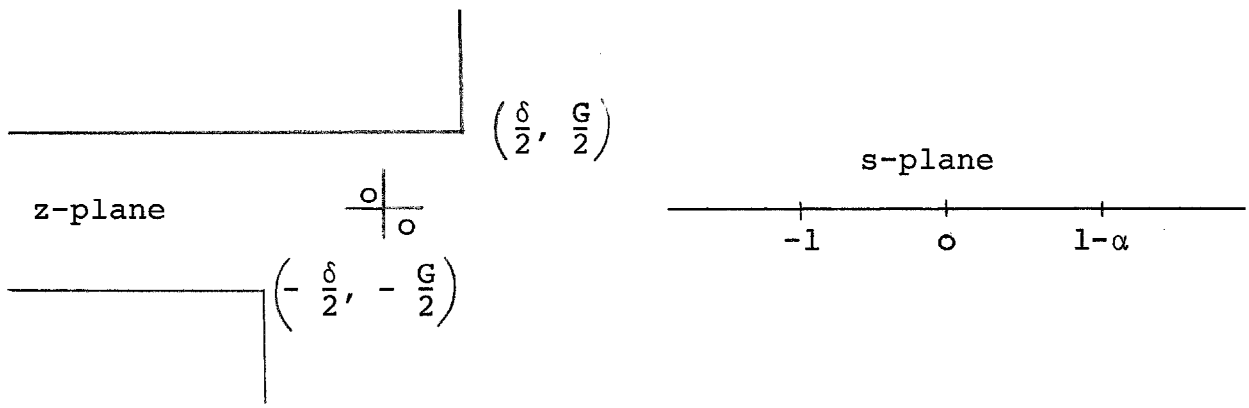
S. C. Snowdon

May 2, 1969

PURPOSE

To find the effect of vertical misalignment and a small machining error in the profile of the booster magnet. Conformal mapping is used to find the solution to two reasonably tractible problems.

VERTICAL MISALIGNMENT



$$z = C_1 \int \frac{(1-\alpha-\alpha s-s^2)^{1/2}}{s} \, ds + C_2 + iC_3 \tag{1}$$

The constants may be evaluated by noticing that when $s = -1$, $z = \frac{\delta}{2} + i\,\frac{G}{2}$ and when $s = 1-\alpha$, $z = -\frac{\delta}{2} - i\frac{G}{2}$. Thus

$$C_1 = \frac{G}{\pi\sqrt{1-\alpha}}\ , \tag{2}$$

$$C_2 = \frac{G}{\pi}\,\ln\left(\frac{1-\frac{\alpha}{2}}{\sqrt{1-\alpha}}\right) \tag{3}$$

$$C_3 = -\frac{G}{2}\left[1 + \frac{\alpha}{\pi\sqrt{1-\alpha}}\ \ln\left(1-\frac{\alpha}{2}\right)\right] , \tag{4}$$

$$\frac{\alpha}{\sqrt{1-\alpha}} = \frac{2\delta}{G} , \tag{5}$$

where

$$z = C_1\left\{ (1-\alpha-\alpha s-s^2)^{1/2} + \frac{i\alpha}{2}\ \ln\ \left[(1-\alpha-\alpha s-s^2)^{1/2} + is + i\ \frac{\alpha}{2}\right] - \sqrt{1-\alpha}\ \ln\left[\frac{(1-\alpha-\alpha s-s^2)^{1/2} + \sqrt{1-\alpha}}{s} - \frac{\alpha}{2\sqrt{1-\alpha}}\right]\right\} + C_2 + i\ C_3. \tag{6}$$

The potential in the s-plane is chosen to be

$$W = \frac{2V_o}{\pi}\ \ln\ s - i\ V_o\ . \tag{7}$$

Hence

$$H^* = i\ \frac{dW}{dz} = i\ \frac{2V_o}{\pi}\ \frac{\sqrt{1-\alpha}}{\sqrt{1-\alpha-\alpha s-s^2}}\ . \tag{8}$$

For small $|s|$

$$H^* = i\ \frac{2V_o}{G}\left\{1 + \frac{i\alpha}{2(1-\alpha)}\ e^{\frac{\pi W}{2V_o}} - \left[\frac{1}{2(1-\alpha)} + \frac{3\alpha^2}{8(1-\alpha)^2}\right] e^{\frac{\pi W}{V_o}} + \quad\right\} \tag{9}$$

and

$$z = C_1\left\{\sqrt{1-\alpha} + i\ \frac{\alpha}{2}\ \ln\ \left(\sqrt{1-\alpha} + i\ \frac{\alpha}{2}\right) + \sqrt{1-\alpha}\ \ln\ s - \sqrt{1-\alpha}\ \ln\ (2\sqrt{1-\alpha})\right\} + C_2 + i\ C_3\ . \tag{10}$$

To first order in α

$$x \rightarrow \frac{G}{\pi}\ \left(1 - \ln 2 + \frac{\alpha}{2} + \frac{\pi U}{2V_o}\right), \tag{11}$$

and on the median plane

$$H_y \rightarrow -\frac{2V_o}{G}\left\{1 - \frac{1}{2}\ (1-\alpha)\ e^{\frac{\pi U}{V_o}}\right\}. \tag{12}$$

Thus

$$H_y \to \frac{2\pi V_o}{G^2}\, e^{-2+2\ln 2+\frac{2\pi x}{G}} \tag{13}$$

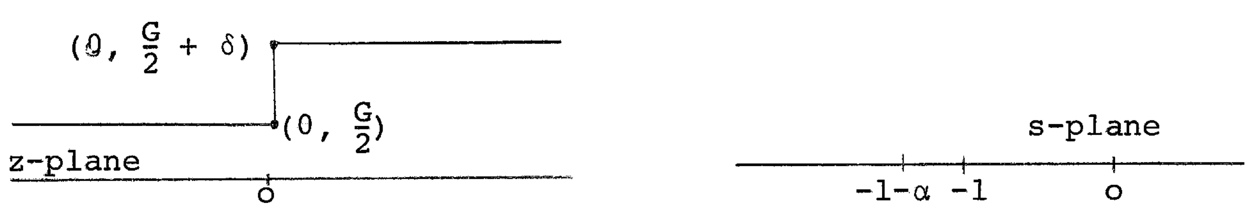
which is independent of α.

Hence from Eq. (13) one concludes that vertical misalignments in rectangularly terminated magnet poles do not have a first order effect on the fields.

PROFILE MACHINING ERROR

The previous problem, while of some interest, may not be directly applicable to the NAL booster magnets since significant edge shimming is utilized to provide the required good field width with a minimum pole width. Thus a relative vertical displacement of the shimming may upset the detailed balancing of the contouring effects. This unbalance will be geometrically evidenced by an excess or deficiency of iron at any particular place on the profile relative to the theoretical requirements. In addition, actual machining errors will give rise to similar excesses or deficiencies. Thus the following problem is felt to contain all the pertinent information relative to the tolerances in the spatial specification of the pole contours.

SPATIAL TOLERANCES



$$z = C_1 \int \left(\frac{s+1}{s+1+\alpha}\right)^{1/2} \cdot \frac{ds}{s} + C_2 \ . \tag{14}$$

which integrates into

$$z = \frac{C_1}{\sqrt{1+\alpha}} \ln \left[\frac{\sqrt{1+\alpha}\left(\frac{s+1}{s+1+\alpha}\right)^{1/2} - 1}{\sqrt{1+\alpha}\left(\frac{s+1}{s+1+\alpha}\right)^{1/2} + 1} \right]$$

$$-C_1 \ln \left[\frac{\left(\frac{s+1}{s+1+\alpha}\right)^{1/2} - 1}{\left(\frac{s+1}{s+1+\alpha}\right)^{1/2} + 1} \right] + C_2 \qquad (15)$$

By inserting into Eq. (15) the conditions, $z = i\,\frac{G}{2}$ for $s = -1$, $z = i\left(\frac{G}{2} + \delta\right)$ for $z = -1-\alpha$, and $z = 0$ for large real positive s, one finds

$$C_1 = \frac{\delta}{\pi} \frac{\sqrt{1+\alpha}}{\sqrt{1+\alpha}-1} \quad , \qquad (16)$$

$$C_2 = i\left(\frac{G}{2} + \delta\right) \quad , \qquad (17)$$

and

$$\sqrt{1+\alpha} = 1 + \frac{2\delta}{G} \, . \qquad (18)$$

The potential in the s-plane is taken to be

$$W = \frac{V_o}{\pi} \ln s. \qquad (19)$$

hence

$$H^* = i\,\frac{dW}{dz} = i\,\frac{V_o}{\pi C_1}\left(\frac{s+1+\alpha}{s+1}\right)^{1/2}. \qquad (20)$$

For $s = \rho$ where ρ is real and positive

$$H_y = -\frac{V_o}{\pi C_1}\left(\frac{\rho+1+\alpha}{\rho+1}\right)^{1/2} \qquad (21)$$

and hence $H_y' = \partial H_y/\partial x$

$$H_y' = \frac{V_o}{\delta^2} \cdot \frac{\pi\alpha}{2} \left(\frac{\sqrt{1+\alpha}-1}{\sqrt{1+\alpha}}\right)^2 \frac{\rho}{(\rho+1)^2} \, . \tag{22}$$

The maximum value of H_y' occurs for $\rho = 1$. Thus, using Eq. (18)

$$H_{y(max)}' = 2\pi \frac{V_o}{G^2} \cdot \frac{\frac{\delta}{G}\left(1+\frac{\delta}{G}\right)}{\left(1+\frac{2\delta}{G}\right)^2} \tag{23}$$

For small $\frac{\delta}{G}$

$$H_{y(max)}' = 2\pi V_o \frac{\delta}{G^3} \, , \tag{24}$$

or, if Δk is the relative gradient $H_y' / (2V_o/G)$

$$\boxed{\Delta k = \pi \frac{\delta}{G^2}} \tag{25}$$

For the parameters of the NAL booster magnets one has:

Allowed Gradient Error (percent)	Tolerance (inches)	
	F	D
±.1	±.00005	±.0001
±1.	±.0005	±.001